



EAST LONDON EXTREME RAINFALL IMPORTANCE OF GRANULAR DATA

LLOYD'S EMERGING RISKS TEAM REPORT

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CONTACT DETAILS

Director, Performance Management

Tom Bolt
+44 (0)20 7327 6700
tom.bolt@lloyds.com

Head of Exposure Management

Paul Nunn
+44 (0)20 7327 6402
paul.nunn@lloyds.com

Emerging Risks Team

Trevor Maynard
+44 (0)20 7327 6141
trevor.maynard@lloyds.com

Gao Yu
+44 (0)20 7327 6641
gao.yu@lloyds.com

EMERGING RISKS TEAM

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The Lloyd's Emerging Risks team maintains a database of emerging risks that is updated regularly through conversations with the Lloyd's emerging risks Special Interests Group, which consists of experts within the Lloyd's market put together with help from the Lloyd's Market Association. The team also maintains contact with the academic community, the wider business community and government. Contact with academics is often facilitated through the Lighthill Risk Network, an organisation that is run as not-for-profit funded by Aon Benfield, Catlin, Guy Carpenter and Lloyd's.

More details can be found at www.lloyds.com/emergingrisks.

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- The Met Office for access to the meteorological archive
- Climate Change Risk Management Ltd for carrying out the analysis presented in this report. More of its work can be found at www.ccrm.co.uk

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EXECUTIVE SUMMARY

- 1. MANY OF THE LONG TERM METEOROLOGICAL STATIONS ESTABLISHED IN THE EARLY 20TH CENTURY WERE CLOSED FROM 1960 ONWARDS.** Only one station, Crossness Sewage Pumping Station, met the criteria of being close to the region of interest. The record began on 1 January 1871. However, the data set was incomplete and alternative data sets were obtained from the Met Office covering the periods of missing record. The station ceased recording in November 2005 and additional records come from three other close by stations. This data was made freely available by the Met Office. We stress our analysis relates to a single weather station with corroborating findings from another location. More analysis would be required to conclude the findings are London-wide, however our results hint that this may be the case.
- 2. FROM 1960 ONWARDS, MORE EXTREME DAILY RAINFALL WAS OBSERVED.** The analysis in this paper shows that extreme daily rainfall became more common from the 1960s onwards. Only one day prior to 1960 recorded rainfall exceeding 40mm, compared to ten days after this period. Analysis of rainfall shows that the frequency of extreme events increased across the board, with the increase being more significant for larger events.
- 3. A DAY WITH 45MM RAINFALL HAD A 30-YEAR RETURN PERIOD BEFORE 1960, AND NOW HAS LESS THAN A SIX-YEAR RETURN PERIOD OF OCCURRENCE.** The pattern of extreme daily rainfall after 1960 is different to the period prior to this. If we compare rainfall with a ten-year return period, we find average intensity post-1960 is 34% greater than the previous period.
- 4. THE TRENDS IN THIS PAPER WOULD BE MISSED UNDER ANALYSIS OF ANNUAL OR MONTHLY RECORDS.** The change in extremes in the daily record is entirely hidden from annual, seasonal and monthly records. This highlights the importance of collecting data and modelling at an appropriate level of granularity.
- 5. WET WINTERS DO NOT TEND TO FOLLOW WET OR DRY SUMMERS.** It is also perhaps surprising that on average the same amount of rain falls in the spring/summer (April-September) months as the autumn/winter. Although, October and November are the wettest months, they are offset by the driest months, February and March.
- 6. THE RESULTS ARE CONSISTENT WITH SOME CLIMATE CHANGE PREDICTIONS BUT NOT OTHERS.** Climate models predict an increase in the frequency and severity of extreme rainfall events; which is what this analysis observes. However, some climate models also predict a shift towards wetter winters and dryer summers – this has not been observed in the records we analysed. Some other climate models do not predict wetter winters, demonstrating the lack of agreement of such models on regional predictions, especially with regards to precipitation. This highlights the importance of not optimising too strongly to a specific model – we must be aware of the uncertainty in model outputs as well as the best estimate predictions.
- 7. PUBLIC AND PRIVATE STAKEHOLDERS NEED TO ENGAGE TO ENSURE APPROPRIATE INFORMATION IS COLLECTED TO ALLOW FURTHER ANALYSIS.** This report shows that climate trends can be hidden unless the data is analysed at an appropriate granularity. Such analysis first requires the data to have been collected over a long period and to be made freely available, so the closing of meteorological stations is of concern. However, it appears that rainfall data is available from a variety of other sources and we hope this can be located, pooled and freely shared in the interests of climate risk management.

INTRODUCTION

There is a scientific consensus (Oreskes 2004) that the mean surface temperature of the Earth has warmed in recent decades, and that the warming amounts to around 0.8°C since the beginning of the 20th Century (IPCC 2007). From this, Goddard Institute of Space Studies (GISS) estimate that 2005 tied with 2007 as the warmest year since reliable instrumental measurements became available, and the first six months of 2010 appear to have exceeded these. Detection and attribution studies show that there is a high probability (at least 90%) that this warming is largely the result of man-made emissions of greenhouse gases (mainly CO₂) in the troposphere and that the amount and rate of warming are outside of the range of natural variation identified during the latter part of the last 11,000 years.

Continued warming is expected to have important consequences for a range of Earth systems (including the atmosphere, cryosphere, oceans, hydrological systems and the biosphere) and there are compelling reasons to expect increases in the magnitude and frequency of some natural hazards such as floods (Huntington 2006), droughts (Mason and Goddard 2001) and landslides (e.g. Fischer et al. 2006). There are also concerns about the stability of several of the large ice sheets on Earth (e.g. Overpeck et al. 2006), as these have the ability to impact upon global sea levels and regulate ocean currents. (See Lloyd's 360 Risk Insight Report "Rapid Climate Change" – www.lloyds.com/360.)

The pattern and extent of future warming has enormously important policy implications for governments and business. The only way in which such predictions can be made is through the use of Global Climate Models (GCMs) and through risk-based analyses.

It is clear that climate change presents a risk to the facilities and infrastructure of national and local authorities. An assessment and analysis of these risks is required to develop a robust risk assessment, mitigation and adaptation programme at regional and national levels.

The objectives of the preliminary analysis presented in this report were to:

- Describe the patterns of daily rainfall in London and compare the early 20th Century record with that of the late 20th / early 21st Centuries.
- Catalogue the occurrence of rainfall events that have resulted in major downpours in the last century.
- Analyse and interpret changes in daily rainfall patterns.
- Analyse trends in annual, seasonal and monthly rainfall patterns.

DATA SETS AND INITIAL ANALYSIS

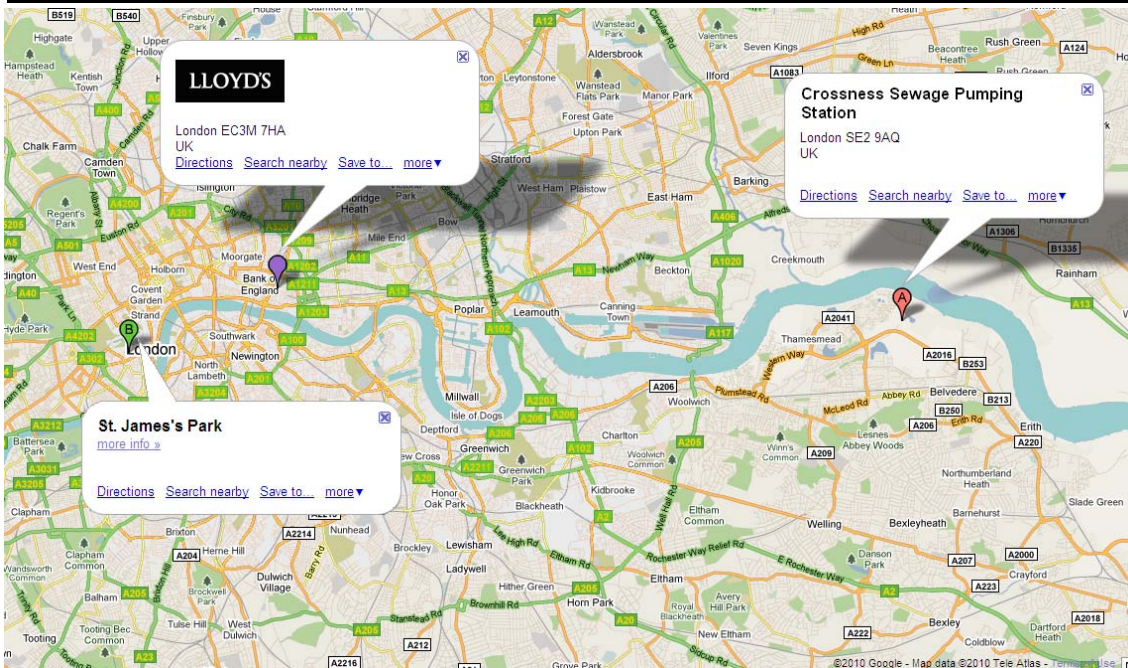
This section details the data sets that were used in this analysis and provides expert views on their quality and suitability. The daily meteorological data analysed for this report were subdivided into two time periods (1915-1960 and 1961-2006) since Foster (1995) has shown, from an analysis of similar daily records in the English Midlands, that major changes in the magnitude and frequency of daily rainfall occurred from around the beginning of the 1960s. In the analysis presented by Foster (1995), the latter period showed significant increases in the number of days with rainfall exceeding 25mm and in the proportion of annual rainfall delivered by these events. The purpose of this subdivision was to establish whether there was any evidence for a change in the magnitude and frequency of rainfall delivered to East London over the last ca.50 years in comparison with the earlier records.

THE MET OFFICE DAILY DATA SETS

Searches of the Met Office archive revealed that many of the long-term meteorological stations established in the UK at the beginning of the 20th Century were closed from the 1960s onwards. In consequence, only one station appeared to meet the criteria of being close to the region of interest (Crossness) being recently operational, and having in its data base ca.100 years of daily data. From the data descriptions provided by the Met Office there was no evidence to suggest that records would have a substantial amount of missing data, although it was anticipated that data sets might have been missing for periods during the First and Second World Wars, which turned out not to be the case.

Daily rainfall data for Crossness Sewage Pumping Station (Figure 1) were provided by the Met Office in paper format. The record began on 1 January 1871. However, the data set was incomplete and alternative data sets were obtained from the Met Office covering the periods of missing record. The data was coded from the paper record into an Excel spreadsheet¹. The spreadsheet contains data from East London Crossness Sewage Station (Station Number 927219) from January 1871- December 2009.

Location of Crossness Rain Gauge (A), St. James's Park (B) and Lloyd's (Figure 1)



¹ Data available at www.lloyds.com/emergingrisks

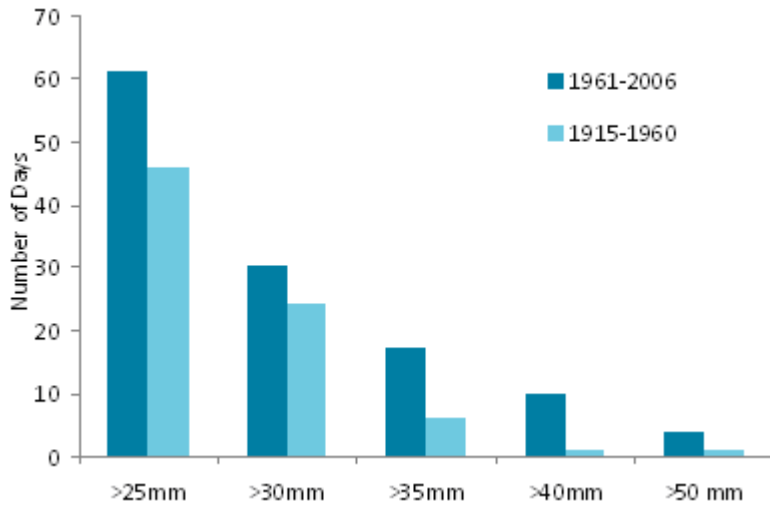
The station ceased recording in November 2005 and additional records come from Deptford Pumping Station, Greenwich (Station number 289101). Small amounts of missing records in the latter half of the period were also in-filled from Deptford, and some additional data were obtained from readings taken at Grove Park Cemetery, Lewisham (Station number 288963) and New Beckenham (Station number 288856).

This report also includes comparative results of rainfall research carried out by the consultancy Climate Change Risk Management (CCRM) in the West London region, using data from a weather station in St. James's Park. These are included to compare the results from two distinctly separate regions in London, giving greater confidence that the high level results may be broadly extrapolated to a wider region.

DAILY RAINFALL

The data was analysed over two separate periods (1915-1960 and 1961-2006), providing two 46 year periods.

Number of days exceeding threshold (Figure 2)



Daily rainfall exceeding 25mm occurred on 107 occasions in the 92 year period, with more in the post-1960 period (61) than in the pre-1960 period (46), (Figure 2).

Figure 3 shows the percentage of increase of various daily rainfall levels between the two time periods. For example, the number of days exceeding 40mm has increased from once between 1915-1960, to 10 occurrences, a 900% increase, between 1961-2006.

Percentage increase in total daily rainfall levels prior against post-1960 (Figure 3)

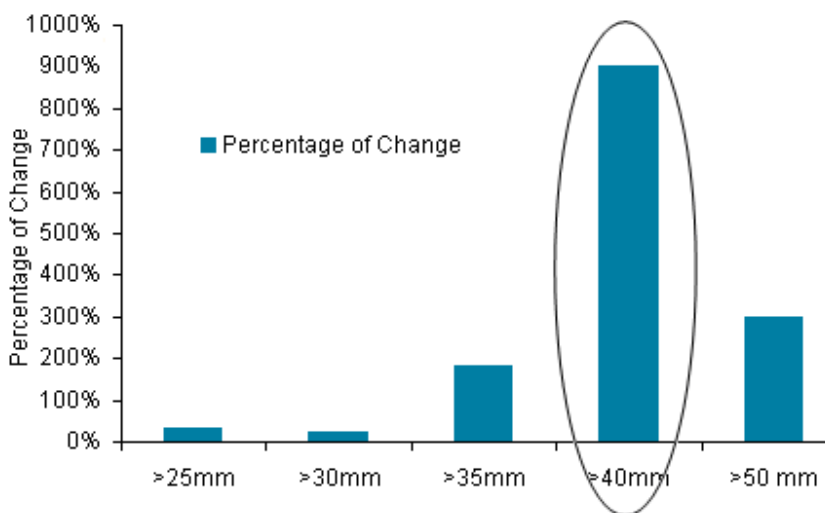


Figure 4 shows a time series of days exceeding 25mm in the record. It is evident from this plot that there are several years in which no days exceeded 25mm in any one year, most notably in the 1930s and the late 1980s, and that there is a tendency for more days exceeding the 25mm threshold to occur after 1960. The mean number of days where rainfall exceeded 25mm in each period shown in Figure 4 illustrates this trend.

Number of days exceeding 25mm (Figure 4)

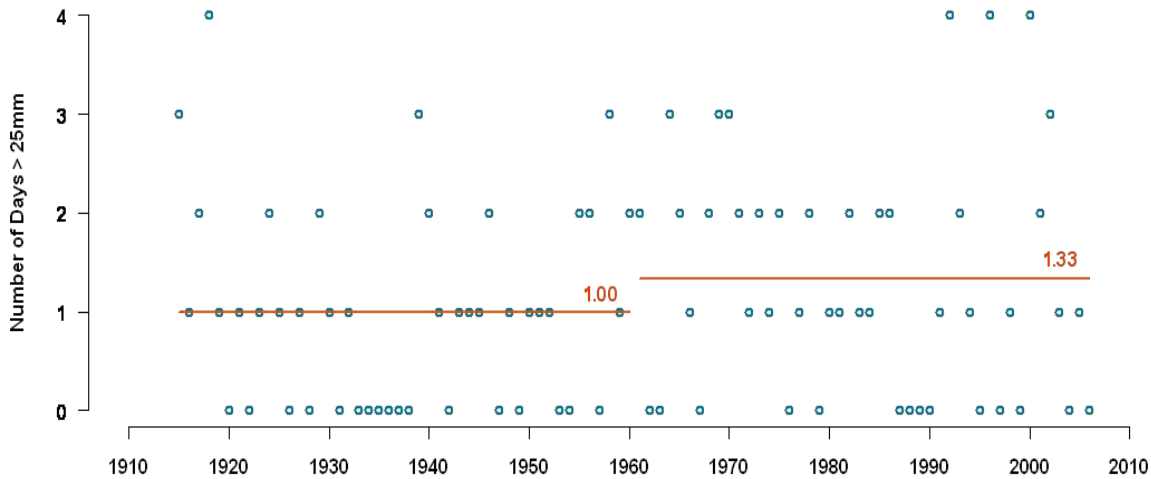
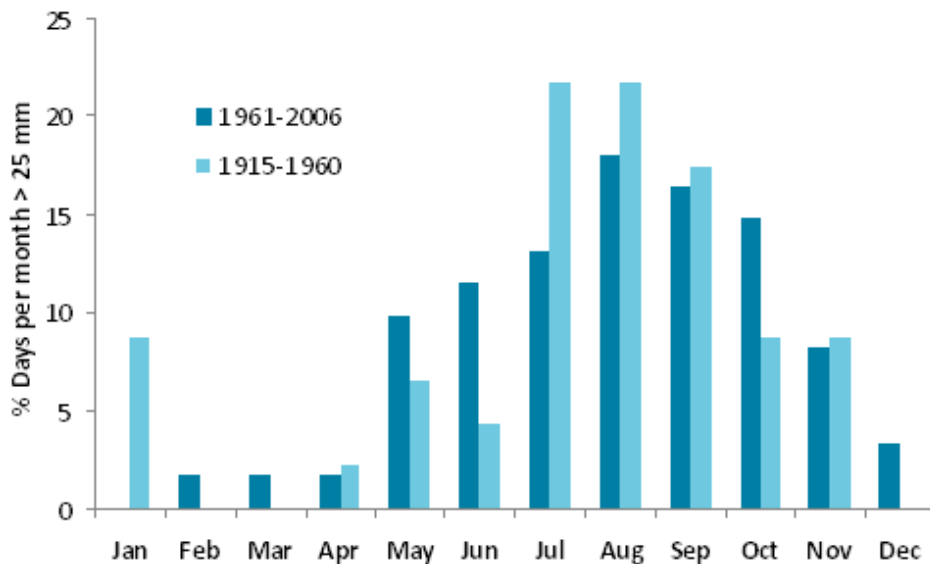


Figure 5 plots the monthly distribution of daily rainfall levels exceeding 25mm expressed as a percentage of the total number of days in the record for the pre and post-1960 periods. The “percentage of days per month” for a specified rainfall threshold (e.g. 25 mm) is the proportion of all extreme rainfall days in a given month.

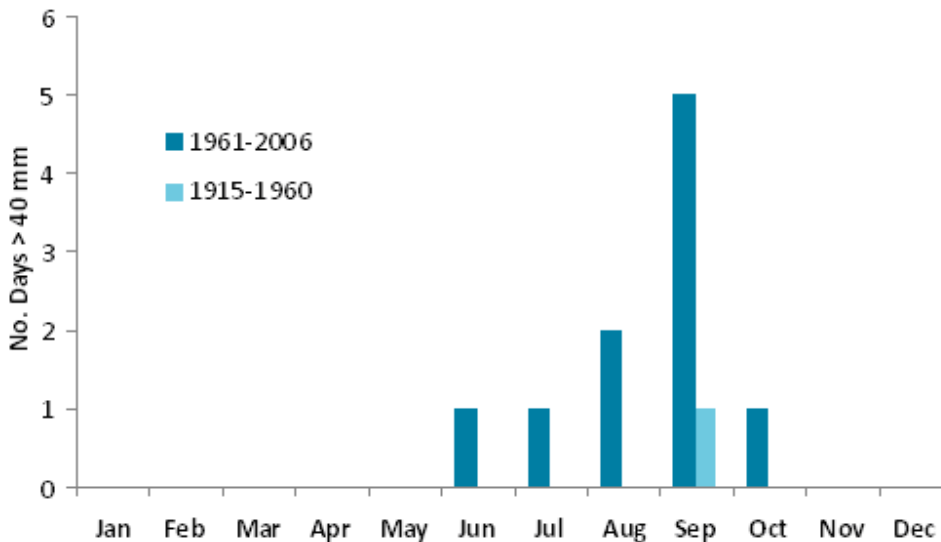
Seasonal distribution of daily rainfalls >25mm (Figure 5)



There is evidence from these plots that the post-1960 period has daily rainfalls above 25mm spread more uniformly throughout the year than in the pre-1960 record.

There are only a small number of days where rainfall exceeded 40mm and therefore Figure 6 plots the number of days rather than the percentages of days delivering >40mm of rain in any one month for the pre and post-1960 period.

Seasonal distribution of daily rainfalls >40mm (Figure 6)



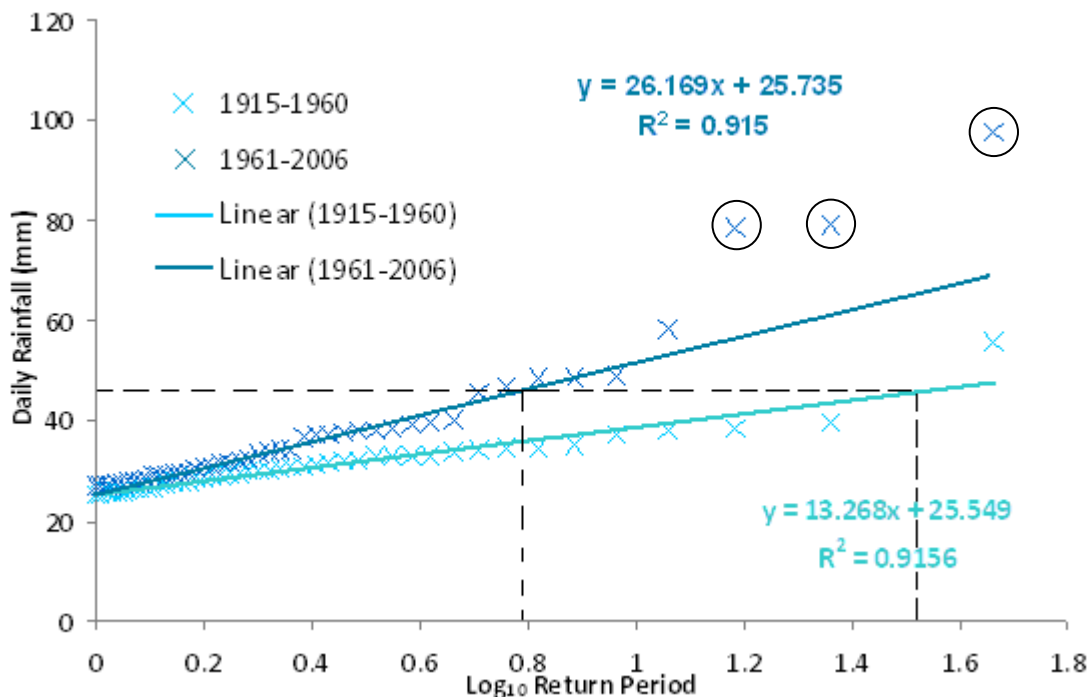
Only one day in the pre-1960 record experienced >40mm of rain, in comparison with ten days in the post-1960 record. These occurred between the months of June and October – 50% of them occurring in September.

Most conventional analyses of daily rainfall magnitude/frequency relationships use a time series based on the maximum daily rainfall (or flood) recorded in each calendar year of record (known as the Annual Duration Series). Where data is missing, or the focus is on the highest daily rainfalls in a time series, an alternative is to take the n highest daily rainfalls corresponding to the number (n) of years of record (The Partial Duration Series), (Shaw, 1994). So, for example, the 1961-2006 record at Crossness is 46 years long and, in order to calculate return periods for this record, the 46 highest daily rainfalls were extracted from the record. The same approach was adopted for the pre-1960 record.

A similar method was used in CCRM's previous research in the West London region. The findings very much coincide with the results shown above. In particular, they found that the number of months where extreme daily rainfall levels exceeding 40mm has increased, and the number of such days has grown significantly between 1915-1960 and 1961-2006.

At Crossness, both 1915-1960 and 1961-2006 show highly significant² ($p < 0.001$) statistical relationships (Figure 7).

Return period analysis (Figure 7)

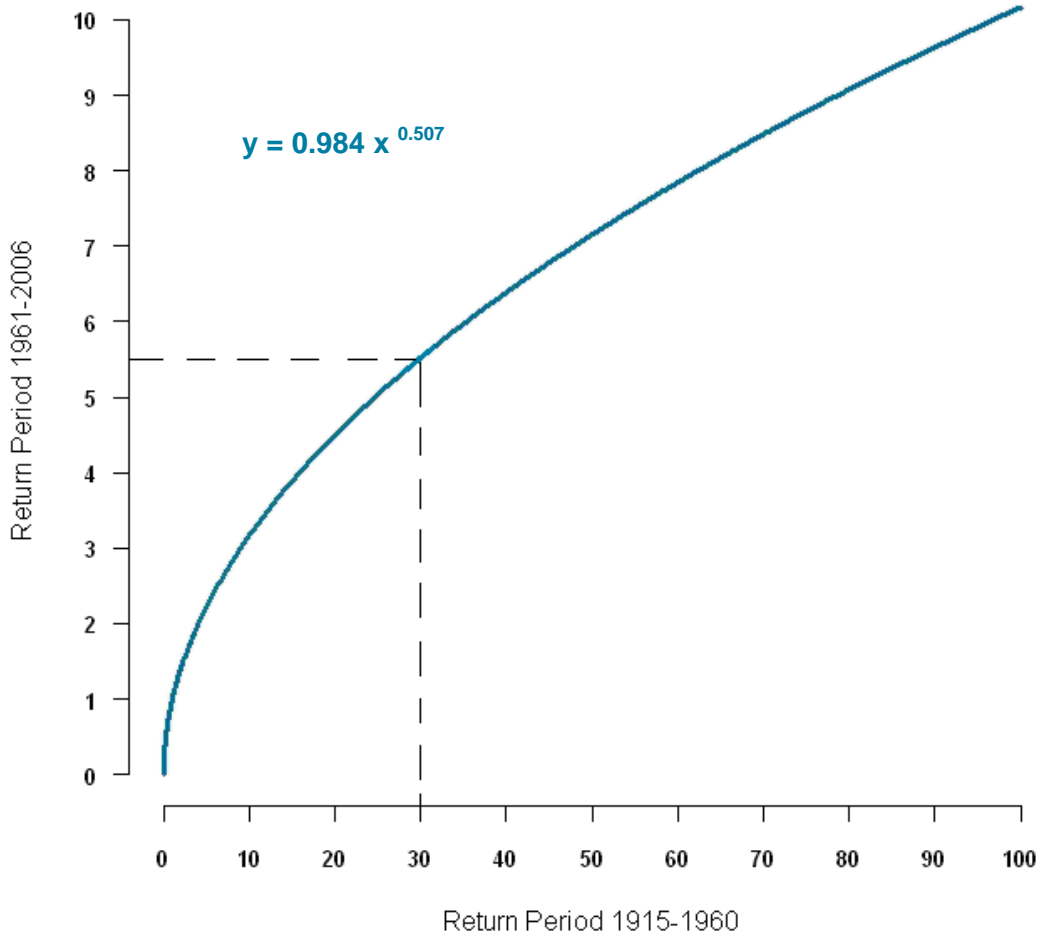


The differences identified in Figure 7 confirm the patterns in extreme daily rainfall illustrated by the previous analysis. Two best-fit lines diverge at longer return periods and show that daily rainfall with a return period of ten years has increased in intensity by 34%.

We can derive how return periods have changed from figure 7. This is calculated by choosing a return period (e.g. $30 \approx 10^{1.5}$ as shown by the dotted line above) and finding the corresponding daily rainfall on the pre 1960 data set (45mm from the light blue line). Then find the return period that this rainfall level corresponds to on the later period ($5.5 \approx 10^{0.75}$ from the dark blue line). Because we read from the light blue line across to the dark blue line we need the fit to the data for the dark blue line to be better at the earlier return periods – for this reason we excluded the three data points circled on the graphic. From the graph above it appears that the gap between really extreme days is even wider than our presentation suggests.

² Please see the appendix which has a note on the calculation of p - values and the T-test.

Return period comparison (Figure 8)



Using the general method described above, we derived the relationship between return periods. As described, a day with rainfall exceeding 45mm had a 30-year return period prior to 1960 whereas, post-1960, it had a 5.5-year return period (illustrated by the dashed line). Days with rainfall occurring with a return period of 100 years pre-1960 had only a 10.2-year return period post-1960.

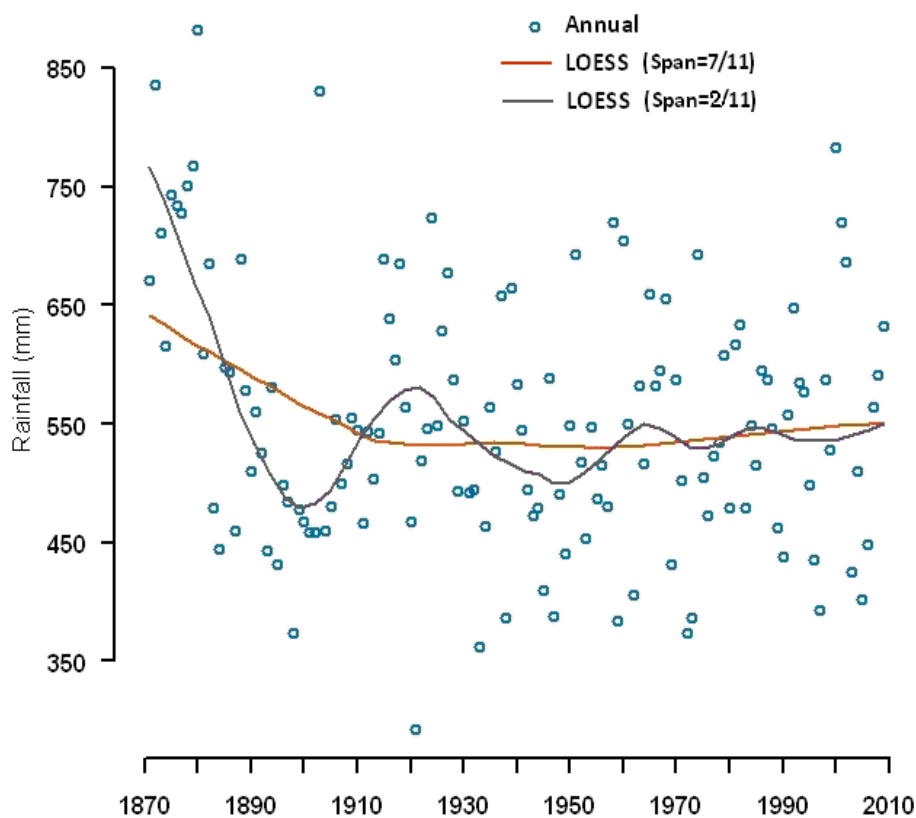
A similar analysis was carried out for CCRM's West London analysis. CCRM found that daily rainfall levels with a return period of 30-years pre-1960, had a 17.7-years return period post-1960. Rainfall days of 48mm intensity have a return period of 30 years in the West London Rainfall report. This shows that the two analyses give the same broad conclusions (extreme events are now more common), but the analysis at Crossness shows a larger change which may be due to regional variability in rainfall and illustrates that the detailed figures in this report are likely to be local in nature.

ANNUAL, SEASONAL AND MONTHLY RAINFALL

Figure 9 shows that annual rainfall for the first two decades of the record was higher than at almost any time during the 20th and 21st Centuries. Over the entire period, annual rainfall averaged 552.2mm with a standard deviation of 107.5mm. A maximum annual rainfall of 882.5mm was recorded in 1880 while the minimum of 292.4mm was recorded in 1921. A smoothed trend line shows a gradual decline in annual rainfall from the 1870s to the 1890s. By the start of the 20th Century, average annual rainfall had decreased by ca.200mm. However, over the 20th and early 21st Centuries there is some periodicity but no statistically significant trends with time.

ANNUAL

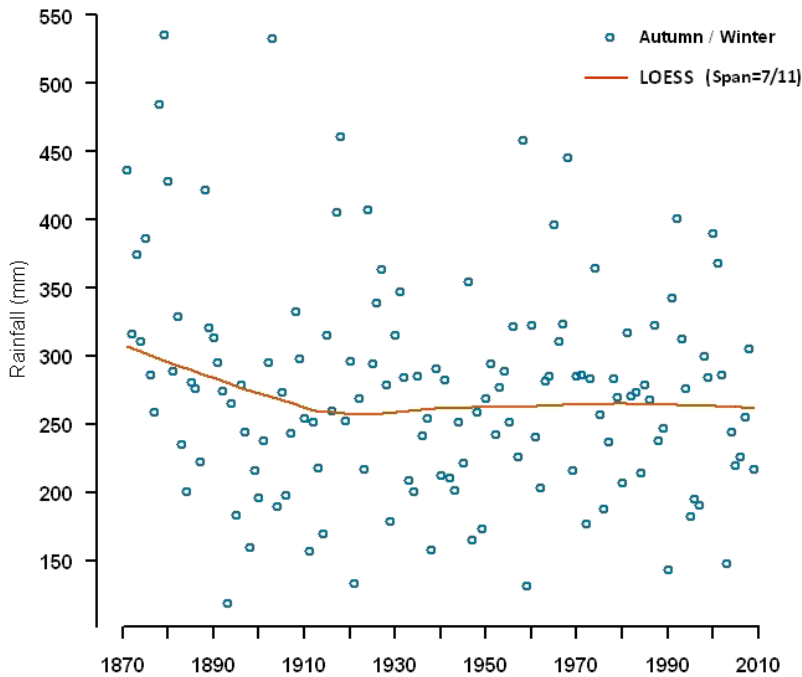
Trends in annual rainfall at Crossness 1871-2009 (Figure 9)



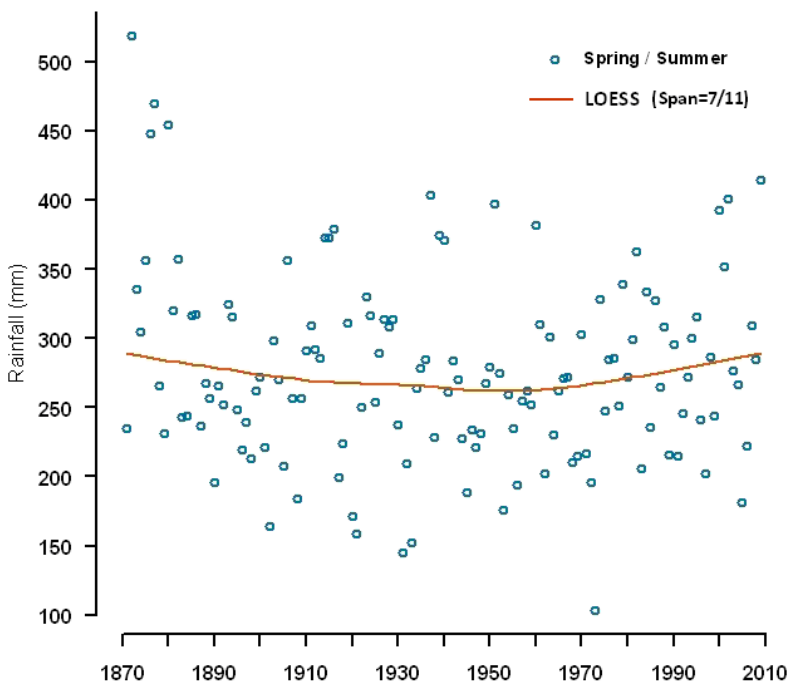
SEASONAL

To analyse seasonal trends, the annual series was subdivided into an autumn and winter data set (October-December and January-March) and a spring and summer data set (April-September inclusive). Figure 10 shows the trend for winter and Figure 11 shows the trend for the summer data series.

Trends in winter rainfall at Crossness 1871-2009 (Figure 10)



Trends in summer rainfall at Crossness 1871-2009 (Figure 11)



Both data sets show similar trends to the annual series. Mean winter rainfall for the whole period is 275.8mm (SD 79.3mm) while for the summer rainfall the long-term average is 276.4mm (SD 67.7mm). There is no statistically significant difference between summer and winter rainfall for the ca.140 year record.

While the patterns appear to be similar, Figure 12 shows that there is effectively zero correlation between winter and summer rainfalls in the same year.

Correlation between winter and summer rainfall (Figure 12)

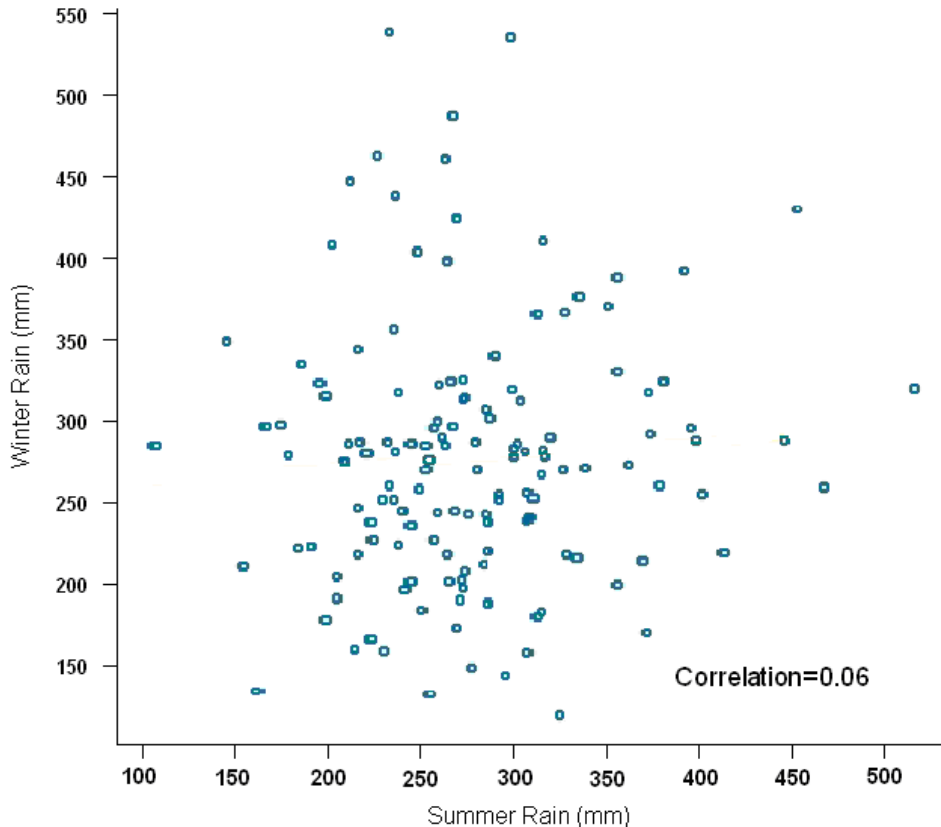


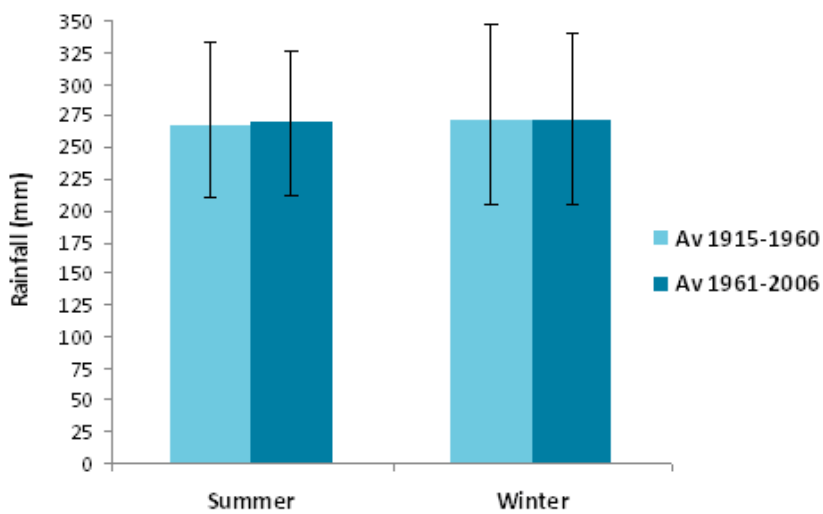
Figure 12 compares the winter and summer rainfall for the time periods corresponding to that used for the daily rainfall analysis (1915-1960 and 1961-2006).

Mean summer and winter rainfalls, with their respective standard deviations, are given in Table 1. There are no statistically significant differences between summer and winter rainfall for each of the periods or between the periods for summer rainfall or winter rainfall. We therefore have no evidence for increases or declines in seasonal rainfall in the latter half of the 20th Century.

Comparison between winter and summer rainfall (Table 1)

		Summer	Winter
Average	1915-1960	267.3	271.3
	1961-2006	268.8	271.5
Standard Deviation	1915-1960	66.6	75.7
	1961-2006	57.9	67.5

Average and standard deviation of summer and winter rainfall (Figure 13)



MONTHLY

Average monthly rainfall for the period 1871-2009 is shown in Figure 14 and indicates a weak seasonality in rainfall delivery. The months of February, March and April are below the average monthly rainfall and October, November and December are above the average monthly rainfall.

Monthly average rainfall and standard deviation 1871-2009 (Figure 14)

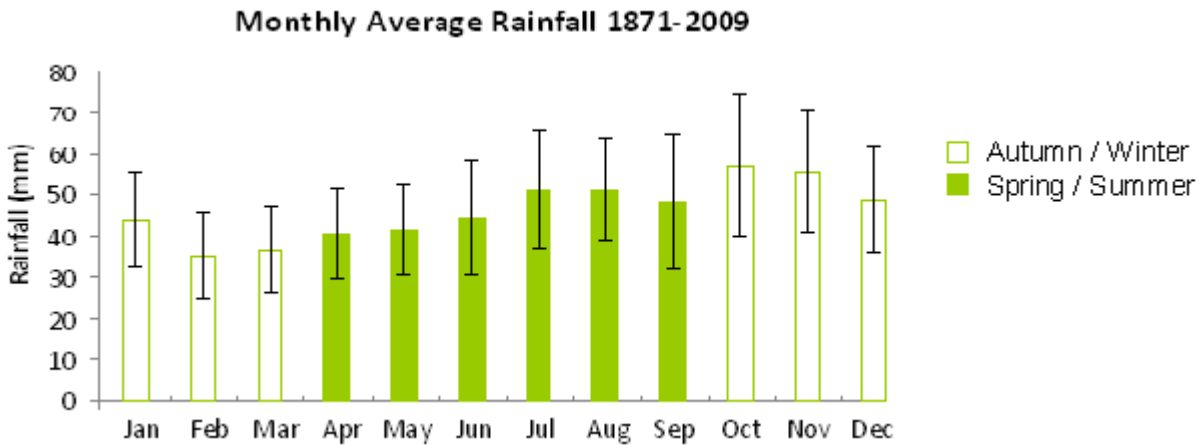
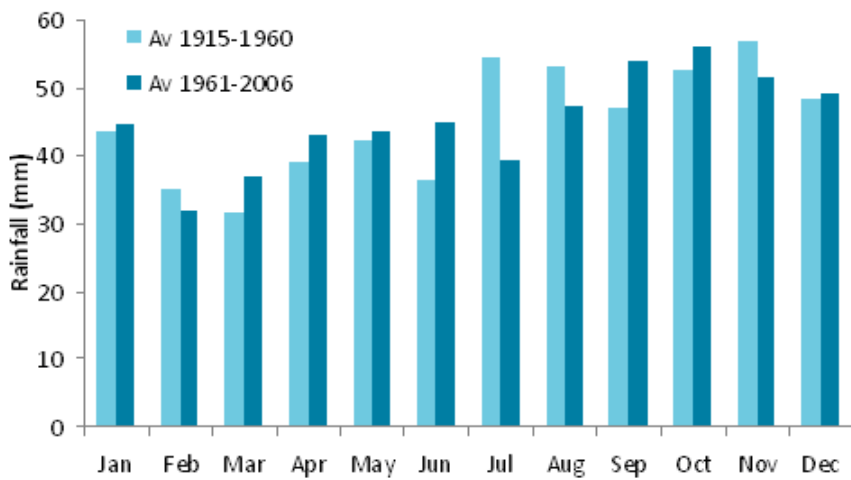


Figure 13 suggests that spring/summer totals are no different to those of autumn/winter. However, Figure 14 shows that autumn has the highest rainfall but this is offset by the lowest rainfall falling in the winter months, particularly during February and March.

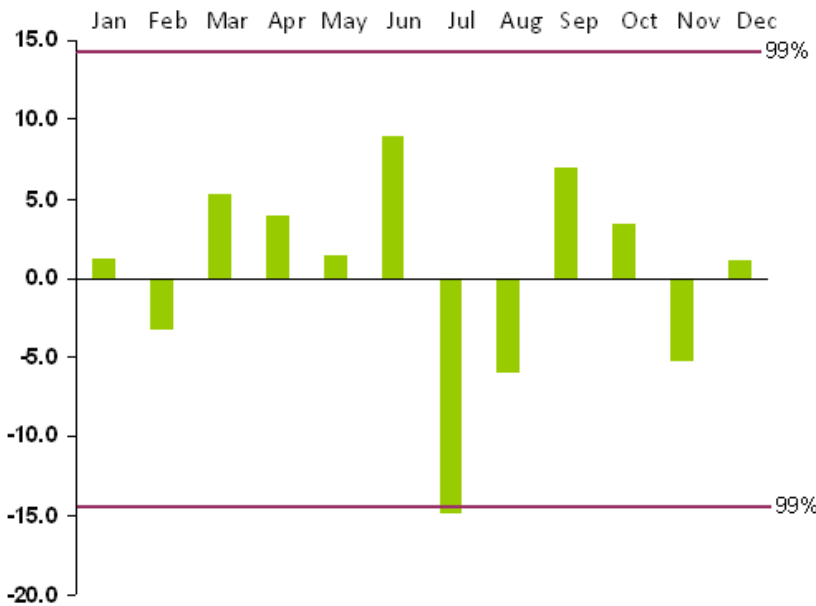
Figures 15 and 16 compare the monthly rainfall between the two observation periods (1915-1960 and 1960-2006). There is little evidence (using the Student's T test³) to suggest that there is any statistically significant change in the seasonal distribution of rainfall between the two periods. The only significant difference ($p < 0.01$) between monthly rainfall in the two accounting periods appeared in July (Figure 15 and Figure 16) when the monthly average fell from 55.3 to 39.3mm between the early and late 20th Century.⁴

The West London analysis finds a similar feature; that July rainfall has reduced by 25%, but other months show nothing significant.

Comparison between mean monthly rainfall amount of 1961-2006 and 1915-1960 (Figure 15)



Changes of mean monthly rainfall amount of 1961-2006 against 1915-1960 (Figure 16)



^{3,5} Please see the appendix which has a note on the calculation of p - values and the T-test.

CONCLUSIONS

This analysis of data from the meteorological station at Crossness illustrates that there have been substantial changes in the pattern of daily extreme rainfall delivery but little evidence that there have been significant changes in annual, seasonal or monthly rainfall, highlighting the need for granular analysis. Unpublished analysis of rainfall data in West London has shown similar results, giving weight to the suspicion that the results may be valid for London as a whole.

The major conclusions with respect to daily and longer-term rainfall patterns are as follows:

DAILY RAINFALL

- The number of extreme daily rainfalls (>40mm) have increased by a factor of ten.
- There has been a change in the seasonal distribution of these extreme rainfalls between the pre and post-1960 records with extreme rainfalls occurring in many more post-1960 months, but with a reduction in the percentage of high daily totals occurring in July.
- September is the most likely month to receive the highest daily rainfall.
- Daily rainfalls at a return period of ten years have doubled in magnitude between the pre and post-1960 periods.

ANNUAL, SEASONAL AND MONTHLY TRENDS

- Using the same subdivision as that used for the analysis of daily rainfall (1915-1960 and 1961-2006) there were no statistically significant differences in annual, seasonal or monthly rainfall with the exception of July monthly rainfall which showed a statistically significant decrease ($p < 0.01$) in the latter period. The shift in the seasonality of extreme daily rainfall appears to be mirrored by the reduction in the average rainfall for the month of July as highlighted above.
- Lack of significant differences was largely a function of the generally high standard deviations for the annual, seasonal and monthly data sets in the pre and post-1960 periods, which makes statistical comparisons less reliable.

The analysis demonstrates that subtle changes in weather patterns in London are unlikely to be observed in coarsely resolved annual, seasonal and monthly series and that analysis of daily rainfall provides the only practical means of extracting subtle changes in weather patterns from existing rainfall records.

Attribution of rainfall trends is not always clear and it cannot be conclusively stated that these changes are a direct consequence of climate change. However, the data here does support the prediction that changing future UK climates are most likely to be associated with a change in the magnitude and frequency of extreme rainfall.

Despite the clear importance of such work for climate change risk assessments, a major issue for future analyses is the loss of records as monitoring stations close. Public and private stakeholders need to engage to ensure appropriate information is collected to allow further analysis. However, it appears that rainfall data is available from a variety of other sources and we hope this can be located, pooled and freely shared in the interests of climate risk management.

SOURCES OF INFORMATION

The following were useful sources of information used when drafting this report. Links are shown for ease of use and were valid at the time of publishing the report:

Fischer, L., Kaab, A., Huggel, C. and Noetzli, J. 2006. Geology, glacier retreat and permafrost degradation as controlling factors of slope instabilities in a high-mountain rock wall: the Monte Rosa east face. **Natural Hazards and Earth System Sciences**, 6, 761-772.

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Shaw, E.M. (1994) **Hydrology in Practice**. London, Chapman and Hall. 3rd Ed.

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APPENDIX

A NOTE ON P - VALUES AND THE T - TEST

A very popular method used for testing statistical significance is Hypothesis testing. Scientists test null hypothesis against the alternative hypothesis. The null hypothesis is commonly the general or default position. For example, the null hypothesis may be that there is no significant change in monthly rainfall amount between 1915-1960 and 1961-2006. The alternative hypothesis would propose a particular relationship between the phenomena, and not necessarily a logical negation of the null hypothesis. In this case, the alternative hypothesis may be there is significant change in some months between the two periods.

Data are assumed to be random. Hence an apparent pattern might just be a consequence of chance. Statisticians use hypothesis testing to rule out chance events. Many statistic distributions admit any value from zero to infinity. Hence, all data sets are possible. However we can calculate how likely the observed data is to have arisen if the null hypothesis were true. This measurement of probability is known as the “p-value”. Assuming that the null hypothesis is correct, the p-value denotes the likelihood of achieving the test statistic at least as extreme as the one that was observed by chance,.

In order to carry out a hypothesis test, a ‘significance level’ must be chosen. The significance level is the level of probability below which allow us to conclude that the null hypothesis is so unlikely to be valid, given the observed data, that we reject it. Very often, scientists choose the significance level to be one of 5% (0.05), 1% (0.01), or 0.1% (0.001). For example, if one chooses to test on a 0.1% significance level was selected, this is equivalent to saying there is one in a thousand probability that the null hypothesis would happen at random. It is important to realise that the chosen level of significance is a matter of convention (this is discussed further in our emerging risks report on Hormone Replacement Therapy where we note that the significance level that is valid for one purpose (e.g. drug prescription) may not be valid if the purpose is different (e.g. horizon scanning)). When the observed p-value is less than the chosen significance level the null hypothesis is rejected.

A T-test is a specific form of hypothesis test where the test statistic is modelled by a Student’s t distribution. The graph below shows the probability density function of a t distribution (PDF), and the formula the PDF. This particular distribution is commonly used when the data being tested has a small sample size which causes the estimated variance to be uncertain (and hence it is inappropriate to assume the observed mean follows a normal distribution).

$$pdf(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\left(\frac{\nu+1}{2}\right)}$$

